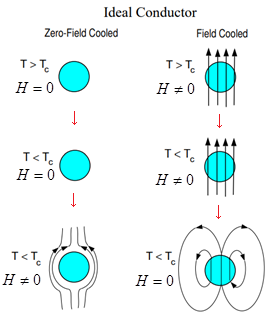
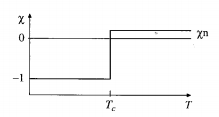
**Meisner Effect**

So if we place a super conductor in a magnetic field, it will form a current to keep the magnetic flux through it *0* (at least below the penetration depth). And if we change the field, the currents will adjust accordingly, to keep all flux out. This is said to be in contrast to merely 0-resistance normal conductors, i.e., ideal conductors, which would circulate current to keep the magnetic flux through it *constant*. I guess this is an indication that superconductivity is a *quantum* effect, not classical. For instance, suppose a conductor transitions to ‘ideal’ at T < Tc. And say we initially had it in zero field. Then if we take our conductor below Tc so it becomes ideal, and apply a magnetic field, it will circulate currents to keep the magnetic flux what it was – namely 0. On other hand, say say we initially had magnetic flux through our conductor at T > Tc. Then we cool it to T < Tc so the conductor becomes ideal, and turn off the magnetic field say, it will circulate currents to keep the flux what it was. In contrast, the superconductor, again, will circulate currents to keep the flux *out* in all cases. This is diagrammed below. Note by H I really mean Bexternal, but they’d be the same (well H = μ0Bexternal) for suitable geometry (solenoidal, and homogeneous substance, etc). And note the field lines are the total B, not just H.



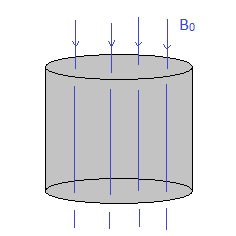
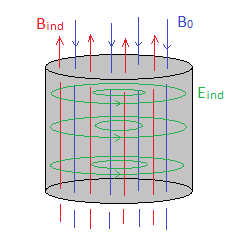
I guess either one – ideal conductor or superconductor – starting from 0 field, would display the following magnetic susceptibility.



I’m guessing χ is positive in the normal state because impurities are interfering with the diamagnetic response – leaving just the paramagnetic response coming from the electron spins?

**Example: Ideal conductor (integral equation approach)**

Can we do an example where we take a magnetic field and slowly turn it on, in the presence of an ideal metal, i.e., one with zero conductivity? We did this example in the EM folder, for metal with non-zero resistivity. We can take the ideal metal cylinder to have radius R and length L. And let B, far from the metal, be some B0(t) = μ0Kf(t), which exponentially slowly increase. In order to have such an external field, we will also need a free (surface) current density Kf(t). I initially left Kf(t) out, but then realized that there were no reasonable solutions to these equations, until I put it back in. So can’t just have a ‘mystery’ free current producing this field B0(t) – it has to be due to the surface current.

If cylinder is ‘infinitely’ long, along the B axis, could we presume induced E field will be simply radially dependent, and oriented along the φ direction? And induced B ought to be as well purely vertical as well? Moreover, we should be able to say that Bind doesn’t depend on r. What are Maxwell’s equations? Our typical equations are:



We have no free charge density. From the continuity equation, this requires at least that ∇·**j** = 0. But we do have free current density, from the surface current wrapping our ideal metal. Also have induced current density coming from electric field driving current around the metal. Normally we’d just say this jind = σE, but an ideal metal has no resistance – infinite conductivity. So let’s reexamine this relationship. Current density is given by:



where I’m using n as the electron number density, rather than ρ, the charge density, basically to distinguish the ρ in Maxwell’s equations, which should be zero thanks to the overall charge neutrality of our metal (and presuming local neutrality too), from the number density, n, of the electrons, which is non-zero regardless. Now normally, for a conductor, we’d say that **j**ind = σ**E**, but for a perfect conductor, rather, there will be no collisions with impurities to impede the electrons and so they will just continually accelerate. Therefore **j** will be given simply by N2L. So we’ll have:



But then we neglect the **j**×**B** guy. Why? I don’t know. Certainly when the current is in its equilibrium state, flowing around the perimeter of the conductor at a constant rate, it is still accelerating centripetally, and **j**×**B** (and maybe some force from the surface of the conductor) would be the only forces capable of supplying it. But I suppose we take these to be negligible. Maybe we’re just keeping terms to first non-zero order in current, **j**, and so maybe our analysis only holds then for small enough currents? We’re ultimately comparing to superconductors, and they *do* have an upper limit on sustainable current, so maybe so. Anyway, then we have:



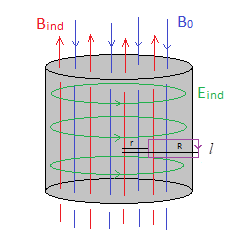
So our equations would be:



ρf = 0 anyway, but we only need the last two equations, which I’m going to transform to integral form:



And let’s apply the first integral equation to a circle, CCW around one of those electric field lines at radius r. And the last integral equation to the purple square of height ℓ, and inner radius r, outer radius R (or slightly larger than R).



Then we have:



(- sgn on Kf comes from RHR stuff, it must be going opposite the ‘positive’ direction defined by our contour) Dividing out the ℓ’s in last equation, and taking note that B0(t) is of course μ0Kf(t), we have:



For what it’s worth, it looks like if we *had* included the j×B term in our ∂j/∂t expression, it would’ve canceled out of our last equation there. If we follow the analysis of the next example, then we’d say….let B0(t) sloooowly increase, which will cause Eind to sloooowly increase, and cause Kind(r,t) to slooowly accelerate, which will induce a slowly changing Bind(r,t). Then this changing Bind(r,t) will make its own contribution to Eind(r,t), and that in turn to Bind(r,t). We can hope that if we do things slowly enough, then the ∂/∂t Eind(r,t) term in the last equation is negligible compared to the others. This term is responsible for EM waves, and so by neglecting it, I think we are basically keeping things in thermodynamic equilibrium. So then we’d have:



Plugging the former into the latter,



Say we suppose Bind(r´´,0) is homogeneous, ‘cause we feel like it:



Kind of sucks. We could take two derivatives w/r to r and turn this into a second order differential equation. But don’t really want to.



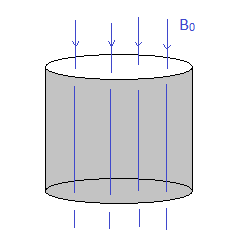
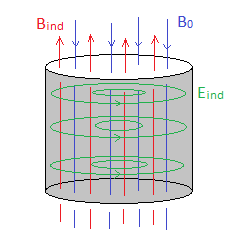
And second derivative,



Yeah no.

**Different Example: ideal conductor (integral equation approach)**

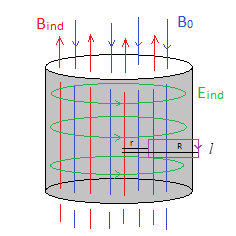
What if do a hollow metal cylinder, with solenoid windings wrapped around it, supplying Kf? Is this any more tractable?

If cylinder is ‘infinitely’ long, along the B axis, we could still presume induced E field will be simply radially dependent, and oriented along the φ direction? A surface current in the metal cylinder will be induced. And this will create a Bind, which ought to be as well purely vertical as well, though possibly radially dependent. Our integral equations are:



And let’s apply the third integral equation to a circle, CCW around one of those electric field lines at radius r. And the last integral equation to the purple square of height ℓ, and inner radius r, outer radius R (or slightly larger than R).



Will observe that Bind should still depend on r, because changing Eind will induce its own contribution to B and so there are three contributions – the free current, the induced current, and the changing field. Then we have:



(- sgn on Kf comes from RHR stuff, it must be going opposite the ‘positive’ direction defined by our contour) I guess I’ll write the induced current as Kind(t) = (e2σ/m)∫0tEind(R,t´)dt´, where σ is the 2D surface number density. Dividing out the ℓ’s in last equation, and taking note that B0(t) is of course μ0Kf(t), we have:



Again, the possible j×B term in ∂j/∂t would’ve canceled out here. This analysis will be a little tendentious. But if we sloooowly increase B0(t), then Eind(r,t) will be small, and so Kind will slowly accelerate, which will slowly increase Bind, which will also contribute therefore to Eind(r,t), which will then also contribute to Bind(r,t). I’m hoping that this last contribution to Bind(r,t) is small enough, that it’s neglectable. This would also make Bind homogeneous. Then we could say,



We’ll have to neglect dEind/dt in order for the lower equation to be consistent. I guess that’s okay, since we’re going slow. This is also the term [see that EM file on metals] which is responsible for generating the cylindrical EM waves, expanding outward from center, that are generated by ‘rapid’ changes in fields . And if we’re supposing that we’re raising the field slowly, keeping things – the EM field – in equilibrium, then we should rule out disturbances like EM waves. So then filling what Kind is,



and setting r = R in top equation:



Filling top equation into bottom one:



This number is:



So practically….



This would predict that the flux through the ring would be constant in time, though not necessarily zero. This is what we were to expect.

**More general analysis: ideal conductor (differential equation approach)**

Let’s go back to ME’s, in metal:



So our equations would be:



Instead of solving for jind in terms of E, we’ll do the reverse, and plug into the third equation. Like we did above, we’ll also neglect the EM wave generating term in the fourth equation, to presume we’re keeping things in equilibrium. And then we’ll have:



and **B**(t) is understood to be the combination of **B**0(t) – coming from **j**f(t) – and **B**ind(t). Maybe take derivative of second guy, so that we have:



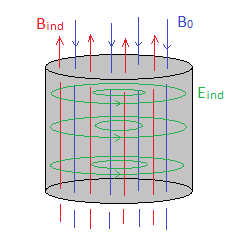
And now let’s confine our attention to the interior of our metal, away from the free current in the solenoid. Then locally, jf = 0, and we can solve for the ∂jind/∂t term in the last guy and plug it into the first guy,



First term is zero because divergence of B is zero. And so then we have:



If we take the geometry from above, and presume magnetic fields going in the z-direction,



then inside the metal, we’d have:



Discounting exponential growth as unphysical, we have exponential decay. And so inside the metal, below the penetration depth,



(This works out to 0.1μm or so) Bz (**B**) must be constant in time. By extension, the magnetic flux there must be constant. But it doesn’t have to be zero per se´.